Maupertuis's Principle of Least Action: Epistemology and Metaphysics

Marco Storni
(Forschungszentrum Gotha der Universität Erfurt)

The paper deals with Pierre-Louis Moreau de Maupertuis's (1698-1759) *writings on the principle of least action* (PLA). *I discuss Maupertuis's philosophical interpretation of the* PLA *by analysing three of his papers* (1740, 1744, 1746)*, in order to assess the overall consistency of his argument. As I shall argue, Maupertuis presents the* PLA *as a universal principle of nature, this being at odds with the empiricist inspiration of his epistemology. Maupertuis's attempt to introduce conceptual elements borrowed from the Leibnizian tradition into a Newtonian-inspired framework creates in fact a tension that is difficult to overcome.*

Keywords: *Maupertuis, principle of least action, Newton, Leibniz, epistemology, metaphysics*

Introduction

In 1740, Pierre-Louis Moreau de Maupertuis (1698-1759) was invited by Frederick II of Prussia to be the chairman of the Berlin Academy of Sciences. Thenceforth, Maupertuis would increasingly become uninterested in regular scientific practice, and inclined more towards philosophical speculation[1]. The development of Maupertuis's interest in philosophical matters roughly coincides with this biographical episode, and is tightly linked to the formulation of the «principle of least action» (PLA)[2]. In what follows, I shall discuss a few epistemological questions linked to Maupertuis's formulation of PLA, most notably the relationship between physical laws and metaphysical principles. This I shall do by relying on three of his papers (Maupertuis 1740, 1744, 1746), which represent three successive steps toward the formulation and philosophical framing of the PLA. I shall start from the core epistemological

[1] See Brown 1963 and Casini 1983.
[2] I am not going here into the scientific details of the PLA. There is abundant secondary literature on PLA: see Brunet 1929, Fee 1941, Feher 1988, Pulte 1989, Radelet De Grave 1998, Leduc 2015, and Lyssy 2015.

distinction between “first-type principles” and “second-type principles,” and proceed by assessing Maupertuis’s attempt to make natural laws – in particular, the laws of geometrical optics – consistent with metaphysical principles of high generality. I shall then inquire into the extent to which Maupertuis’s argument is indebted to Leibniz’s discussion of the laws of optics, and end with a critical presentation of Maupertuis’s own proof for the existence of God.

1. *First-Type Principles* versus *Second-Type Principles*

At the outset of the *Loi du repos* (Maupertuis 1740), Maupertuis distinguishes between two different kinds of principles on which all the sciences[3] are grounded.

> If the sciences are grounded on certain simple and immediately clear principles, on which all the truths that are the object thereof depend, they have yet other principles, less simple indeed, and often difficult to discover, but which once discovered, are of very great utility. These are in some way the laws Nature follows in certain combinations of circumstances, and which teach us what it will do on similar occasions (Maupertuis 1740, 170).

Here, Maupertuis formulates the core epistemological distinction between “first-type principles” and “second-type principles”. Second-type principles are «less simple» and «difficult to discover», and coincide with particular laws of nature (e.g. the principle of the lowest centre of gravity in statics and that of the conservation of living forces in dynamics[4]), whereas first-type principles are «simple and immediately clear». If the status of second-type principles is sufficiently clear, thanks to the examples Maupertuis himself provides, the status of first-type principles remains on the contrary vague[5]. Scholars have suggested various interpretations for first-type principles: they might *e.g.* be general metaphysical principles[6], or physical axioms of the mathematical sciences[7].

Things, however, get trickier when Maupertuis specifies other features of both kinds of principles, in particular concerning their demonstrability.

> Principles of the first type hardly require any demonstration for their evidence is clear to the mind as soon as it examines them. The principles of the second type, however, are not

[3] Not only mechanics, as Marco Panza suggests (Panza 1995).
[4] These are the examples Maupertuis himself mentions.
[5] See Van den Abbeel 2017, 61-63.
[6] Beeson 1992, 165.
[7] Terrall 2002, 175.

> susceptible of physical demonstration in the strict sense, because it is impossible to go through all the cases in which they occur (Maupertuis 1740, 170).

It is impossible to provide any demonstration of first-type principles, as the knowledge of them we have is intuitive[8]. Second-type principles, on the contrary, are demonstrable, but are not so *a pedibus usque ad caput*. A few lines later Maupertuis comments further on this point:

> These principles have never been given a general demonstration in the strict sense; but no one who is accustomed to study the sciences, and who knows the force of induction, will ever doubt their truth (Maupertuis 1740, 170).

Second-type principles are demonstrated according to the methodological lines Newton gave in the *Principia* (1687) and in the *Opticks* (1704). As the fourth *Regula philosophandi* reads,

> In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions. This rule should be followed so that arguments based on induction may not be nullified by hypotheses (Newton 1999, 796).

And again, in the *General scholiu*[9]: «In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction» (Newton 1999, 943). Maupertuis stresses the reliability of Newton's inductive method, although this method provides no complete demonstration for the propositions at stak[10]. Our understanding is too weak, and our knowledge too scanty, to enable us to follow the whole chain of reasons and causes[11]. If we could do so, our demonstrations of second-type principles would be completely a priori:

> As of a priori demonstrations of these kinds of principles, it seems that physics is not able to provide them; they [a priori demonstrations] seem to belong to some superior science (Maupertuis 1740, 170).

[8] It is hard to see, then, how first-type principles could be identified with axioms of Newtonian mechanics (*axiomate sive leges motus*), which seem to require a stronger foundation than mere intuition.
[9] Published for the first time as an appendix to the 2nd edition of the *Principia* (1713).
[10] On Newton's inductivism, see Ducheyne 2012, 107-169, and Belkind 2017.
[11] Sources for this claim are Locke and the sceptical tradition: see Tonelli 1987.

Given the contrast between first-type and second-type principles which Maupertuis had established at the outset, it seems not unreasonable to suppose that the former might indeed be a priori. Their simplicity does not allow for demonstration though, as Maupertuis explicitly states. The intuitive status of first-type principles might likewise shed light on the meaning of the somewhat obscure expression «superior science». If a science superior to physics exists, it might be one that works entirely with a priori intuitive principles, *i.e.*, with first-type principles.

The epistemological discussion carried out in the *Loi du repos* is relevant for it provides conceptual distinctions which will be useful in investigating Maupertuis's effort to integrate physics with the «higher sciences» (metaphysics, theology). Moreover, the discussion of first-type and second-type principles, a priori and a posteriori demonstrations, and of a «superior science» suggests that – as David Beeson correctly stresses – «Maupertuis in this paper takes a step towards Leibnizian architectonic reasoning» (Beeson 1992, 167).

2. *Optics and Metaphysics*

2.1. PLA

The 1744 paper *Accord de différentes lois de la nature qui avaient jusqu'ici paru incompatibles* is well-known for it includes the first clear formulation of the PLA. Although the argument deals with optics, this subject is for Maupertuis no more than «a memorable example» (Maupertuis 1744, 417), useful to discuss a general philosophical point.

> We should not expect that the different means we have to increase our knowledge should lead us to the same truths, but it would be shocking to see that propositions which philosophy presents as fundamental truths, were in fact contradicted by geometry or algebra (Maupertuis 1744, 417).

Maupertuis introduces here a first assumption: the truths which philosophy and the sciences – represented here by mathematics – teach should not be at odds with each other (the "truth-agreement" postulate). From this assumption a first problem arises, which the formulation of the PLA should come as an answer to: although undeniably true, the "truth-agreement" postulate seems to be contradicted by a few counterexamples, of which optics is perhaps the most striking.

There is yet another assumption grounding Maupertuis's discussion, which, however, is made explicit only at the end of the paper. From this second postulate another assumption follows which Maupertuis's argument is meant to ground. God exists, and has organised the world harmoniously. This harmony consists in the fact that a blind mechanism – the laws of nature, *viz*. second-type principles – executes the orders of a superior intelligence. The PLA will thus prove that it is good to employ *final causes* in the natural sciences[12].

I have mentioned that the discussion carried out in the *Accord* concerns optics, first and foremost because it provides a "memorable" counterexample to the "truth-agreement" postulate. Why is this so? Maupertuis believes that, whereas the first two fundamental laws of geometrical optics – (1) light, in a uniform medium, moves in a straight line, and (2) the angle of incidence is equal to the angle of reflection (the "law of reflection") – are philosophically intelligible, for they have evident mechanical analogies (the law of inertia and elastic collision), the third law – the "Snell–Descartes law" or "law of refraction" – is not[13]. Such a law states that, in any refraction, the ratio between the sine of the angle of incidence and the sine of the angle of refraction is a constant determined by the mediums involved. This law lacks intelligibility because, as Maupertuis stresses, «when light passes from one medium to another, the phenomena are very different than when a ball passes through different mediums» (Maupertuis 1744, 419)[14].

Maupertuis distinguishes three interpretative options for the laws of optics, corresponding to three emblematic figures of seventeenth-century science: (1) A mechanical explanation (Descartes); (2) A mechanical explanation, which resorts to an additional non-mechanical assumption (Newton); (3) A non-mechanical explanation (Fermat). The accounts provided by both Descartes and Newton are immediately dismissed. If Descartes's account is not physically accurate – «the way in which this great philosopher tried to explain this phenomenon is imperfect» (Maupertuis 1744, 419-420) – the same, however, does not apply to Newton's and the Newtonians' account. Alexis Clairaut, *e.g.*, in his Newtonian-inspired theory of light,

[12] As d'Alembert writes in the article «Action» of the *Encyclopédie*, Maupertuis' cosmology is based on the attempt to "ally the metaphysics of final causes to the most fundamental truths of mechanics" (d'Alembert 1751, 119).

[13] For an account of modern discussions on optics, see Sabra 1981.

[14] In some sense, one should seek the "sufficient reason" for that law to be. As I will show, the PLA is characterized in much the same way as Leibniz's principle of sufficient reason. See Terrall 2002, 177-178.

«deduces the phenomena of refraction with the same clarity as all the other subjects he deals with» (Maupertuis 1744, 420). Although not explicitly stated, the problem with the Newtonians seems to be philosophical: Maupertuis seems indeed to assume that the Newtonian theory provides no appropriate metaphysical framework for physics[15].

Fermat's method is therefore preferable, as it «seeks the explanation of these phenomena in a wholly different and purely metaphysical principle» (Maupertuis 1744, 420). Fermat formulates, on the basis of empirical evidence, a most general principle: «*Nature, in the production of its effects, always acts in the simplest way*» (Maupertuis 1744, 421). Fermat generalizes this principle from optics to the whole of physics, arguing that all natural movements follow the shortest-path and shortest-time law. The metaphysical principle of economy grounds for Fermat the phenomenal level: both the «direct and reflexed movements of light – Maupertuis stresses – […] seem to *depend on* a metaphysical law» (Maupertuis 1744, 421; my emphasis).

Maupertuis agrees with Fermat on the necessity to ground physical laws on a "higher" principle, but disagrees on the specific contents of Fermat's argument. Fermat's principle, according to Maupertuis, is incorrect, for the physical data from which it is deduced are not accurate. Fermat's mistake is in fact to assume that light moves slower in denser mediums, and quicker in less dense ones. Maupertuis agrees with Descartes (and Leibniz) that the converse is rather true. The refraction of light, therefore, does not happen according to the shortest-path and shortest-time law. The metaphysical principle grounding the laws of optics is another one, namely that according to which the «quantity of action» is minimized, meaning the sum of the distances covered, each multiplied by the velocity with which the body travels through it. Only at this point do we find the first explicit formulation of the PLA: «It [light] takes a path that has a real advantage: *the path it takes is that whereby the quantity of action is the least*» (Maupertuis 1744, 423). «Once the true principle discovered», Maupertuis concludes, «all laws of light can be deduced from it» (Maupertuis 1744, 425).

2.2. *Epistemology and Metaphysics*

[15] When presenting Fermat's position, Maupertuis says that Fermat rejects the Newtonian explanation, «although it is known that this principle [attraction] was neither unknown nor distasteful to him» (Maupertuis 1744, 420). In 1744, Maupertuis's attitude towards Newton remains ambiguous; in 1746, on the contrary, Maupertuis expressly criticizes Newton and the Newtonians for their physicotheology.

The *Accord* provides further elements to pursue the discussion of epistemology started with the *Loi du repos*. In the above-quoted passage stating that «it would be shocking to see that propositions which philosophy presents as fundamental truths, were in fact contradicted by geometry or algebra» (Maupertuis 1744, 417), Maupertuis makes a clear-cut distinction between philosophical and scientific truths. Philosophy is about «fundamental truths», whereas the sciences are confined to contingent truths. According to Maupertuis, mathematics and geometry also deal with natural objects, at a higher level of abstraction than physics, and their conclusions are therefore contingent[16]. *Prima facie*, this opposition seems to reproduce that between a «superior science», including most general truths, and the other sciences, constructed with second-type principles.

The discussion of optics and metaphysics, however, poses some challenges to this epistemological picture. In some passages of the *Accord*, particular scientific truths are said to depend on metaphysical laws – «the direct and reflexed movements of light […] seem to depend on a metaphysical law» (Maupertuis 1744, 421) –, whereas elsewhere it rather seems that scientific laws are the basis for the elaboration (or modification) of metaphysical laws – «on the basis of this fact, *that light moves quicker in denser mediums*, Fermat's and Leibniz's [metaphysical] construction is destroyed» (Maupertuis 1744, 422). It seems therefore reasonable to raise the question whether metaphysical principles ground physical laws, or the other way round. At the end of the paper, Maupertuis gives indications for solving the puzzle. To find «the causes of physical effects», we can either «[calculate] the properties of bodies», or «[search] what it was more appropriate to make them [the bodies] do» (Maupertuis 1744, 425-426). These two methods roughly correspond to induction and deduction, *i.e.*, bottom-up and top-down relations. *De facto*, Maupertuis says, since «the first method [induction] is within our reach, but it does not lead us very far», and «the second [deduction] sometimes misleads us, for we do not know which is the purpose of nature», we should «employ both methods[:] let us calculate the movements of bodies, but also consult the design of the Intelligence making them move» (Maupertuis 1744, 426). Our understanding is too weak and our knowledge too scanty to enable us to follow the entire chain of causes (either bottom-up, or top-down).

[16] So Maupertuis will argue in the *Examen philosophique de la preuve de l'existence de Dieu employée dans l'*Essai de cosmologie (Maupertuis 1756, 393-394).

There is no logical *primum*; the whole research relies on an imperfect empirical procedure.

This is how things are *de facto*. Such an explanation, however, does not provide any answer to the above-raised question whether metaphysical principles ground physical laws, or the other way round. How should things be *de jure*, then? The PLA seems to be formulated on an inductive basis. The research on optics provides evidence to formulate a general conclusion, grounded on phenomena, which is therefore a posteriori. This, of course, would make of the PLA a second-type principle, just like Newtonian attraction. The PLA, however, is clearly put on a different qualitative level than the laws of optics: it is hard to see, then, how induction can allow for such a qualitative passage. Moreover, Maupertuis argues that the PLA is a most general principle. As he writes in a further work (*Letters*, 1752): «I have showed that all laws of movement are grounded on the principle of least action: […] attraction itself depends on the principle of least action […]: it makes the bodies move as they should in order to obey this *universal law of nature*» (Maupertuis 1768b, 288-289; my emphasis). According to the *Loi du repos*, induction cannot provide general (*i.e.*, complete) demonstrations. If the PLA is a «universal law of nature» *stricto sensu*, it should therefore be a first-type principle[17]. Maupertuis's ideas – at least regarding the situation *de jure* – are unclear. All the more so, if one considers that the PLA will become the basis of Maupertuis's metaphysics, whose identification with a «superior science» looks in fact plausible.

3. *Leibniz and Teleology*

Maupertuis's argument is deeply indebted to Leibniz's teleological account of the laws of optics. Although Maupertuis never explicitly quotes Leibniz's work on optics, he was certainly acquainted with Leibniz's optical theories, which he fleetingly refers to[18]. Most importantly, the argument presented in the *Accord* is strikingly similar to

[17] However, it is hard to see how the PLA might be «simple and immediately clear», and how it would «hardly require any demonstration» (Maupertuis 1740, 170) for its self-evidence.

[18] Maupertuis's knowledge of Leibniz's optics is indirect. As he says in the version of the *Accord* published in the 1756 edition of his *Works*: «When I read this paper [the *Accord*] in the Paris Academy of Sciences, I did know Leibniz's work on optics only by what de Mairan says in his paper on the reflection of light, *Mémoires de l'Académie de Paris*, year 1723» (Maupertuis 1768c, 23).

Leibniz's. A comparison between the two accounts might therefore be useful for clarifying the few difficulties in Maupertuis's argument[19].

Although the only text by Leibniz on optics available to Maupertuis would have been the *Unicum opticæ, catoptricæ, et dioptricæ principium* (*A Unitary Principle of Optics, Catoptrics, and Dioptrics*), published in 1682 on the *Acta Eruditorum*, I shall rather focus on the *Tentamen anagogicum, or An Anagogical Essay in the Investigation of Causes*, written in 1695, but published in the course of the nineteenth century. Leibniz advances similar arguments in both papers; the *Tentamen anagogicum*, however, is much more detailed on the relationship between mechanical laws and teleological principles, and for this reason more relevant to my study.

In the *Tentamen anagogicum*, Leibniz attempts to reconcile Descartes's mechanical account of the laws of optics – particularly the law of refraction – with Fermat's teleological account. The Cartesian objections to Fermat's argument (light moves faster in denser mediums) are overcome by means of the «Most Determined Path Principle» (MDPP).[20] A ray of light, Leibniz contends, always moves along the most determined path, which does not necessarily imply the minimization (or maximisation) of some quantity. Leibniz's principle just states that «the easiest path would prevail among all the possible competing rays» (Leibniz 1989, 479), *ease* being defined as «the quantity obtained by multiplying the distance of the path by the resistance of the medium(s)» (McDonough 2009, 512). By means of this general principle, Leibniz can easily derive the "Snell–Descartes law" by means of differential calculus and basic trigonometry[21]. He thus manages to preserve Fermat's teleological approach to optics – MDPP being a teleological principle – adjusting Fermat's principle so as to make it consistent with Descartes's mechanical account of phenomena.

From the epistemological point of view, Leibniz acknowledges the fruitfulness of purely mechanical explanations: efficient causes alone can adequately describe what goes on in the physical world. Mechanical explanations, however, are *per se* insufficient and should be integrated with teleological principles: «The laws of motion cannot be explained through purely geometric principles or by imagination alone» (Leibniz 1989, 478). This is because, although mechanical causes can explain all

[19] There is an abundant literature on Maupertuis and Leibniz. With particular reference to the PLA (and to the debates over this principle), see Kneser 1929, Brunet 1938, Bachelard 1961, Goldenbaum 2016.
[20] See McDonough 2009, 511-512.
[21] For the details of this derivation, see McDonough 2009.

physical phenomena, the ultimate principles of mechanics cannot themselves be explained mechanically, and need teleological principles in order to be grounded.

> All natural phenomena could be explained mechanically if we understood them well enough, but the principles of mechanics themselves cannot be explained geometrically, since they depend on more sublime principles which show the wisdom of the Author in the order and perfection of his works (Leibniz 1989, 478).

The teleological account of natural phenomena has for Leibniz a twofold function[22]. (1) It relates physical nature to God's intelligent design. In corporeal nature, alongside the mechanical dimension, which Leibniz calls «the realm of power, according to which everything can be explained *mechanically* by efficient causes», there is another (and superior) dimension that he calls «the realm of wisdom, according to which everything can be explained architectonically, so to speak, or by final causes» (Leibniz 1989, 479). In nature, therefore, finalistic laws operate as a mark of the divine organization of the world. (2) Leibniz's argument displays the inventive role of final causes in physics[23]. Teleology is for Leibniz a proper method for physical investigation: in the case of optics, «we may well believe that we should not have had this beautiful discovery [the law of refraction] so soon without the method of final causes» (Leibniz 1989, 480)[24]. Optics thus provides an interesting case study to demonstrate that teleology might work as a powerful explanatory device.

> So we begin here to show that no other reason can be given for the laws of nature than the assumption of an intelligent cause. Or we show also that in the investigation of final causes

[22] On this point, see Duchesneau 1993, 263-264.
[23] In the *Unicum opticæ* too, Leibniz insists on this very point: «And so those who reject final causes in physics with Descartes err greatly – not to speak more harshly – since even besides the admiration of divine wisdom, they would also supply to us the most beautiful principle for discovering some properties of those things whose interior nature is still not so clearly known to us that we would be able to use proximate efficient causes and explain the machines which the Creator employed in order to produce those effects and in order to obtain his ends». Translated by Jeffrey K. McDonough. http://philosophyfaculty.ucsd.edu/faculty/rutherford/Leibniz/unitary-principle.htm#note1. Accessed 16 January 2018.
[24] Leibniz thinks that Snell had first discovered the law of refraction, and that Descartes contributed little to the discovery. «It is known that Willebrord Snell, one of the greatest geometricians of his time and well versed in the methods of the ancients, invented it, having even written a work which was not published because of its author's death. But since he had taught it to his disciples, all appearances point to the conclusion that Descartes, who had come to Holland a little later and who was most interested in this problem, learned it there. For the way in which Descartes has tried to explain the law of refraction by efficient causes or by the composition of directions in imitation of the reflection of bullets is extremely forced and not intelligible enough. To say no more about it here, it shows clearly that it is an afterthought adjusted somehow to the conclusion and was not discovered by the method he gives». (Leibniz 1989, 479-480).

> there are cases in which it is necessary to consider the simplest or most determinate, without distinguishing whether this is a maximum or a minimum […]. Certain new general theorems can be derived which apply equally to refraction and reflection (Leibniz 1989, 484-485).

As we see, Leibniz insists on the heuristic role of teleological principles. The inventive power of such principles is well displayed by the case of optics, where MDPP would have not been found without recourse to teleological reasoning.

Leibniz, however, wants to go as far as to imply that teleological principles might have a heuristic function in all fields of knowledge. As he writes in the *Definitiones cogitationesque metaphysicae* (1678-80): «All the phenomena of nature can be explained solely by final causes, exactly as if there were no efficient cause; and all the phenomena of nature can be explained solely by efficient causes, as if there were no final cause»[25]. According to Leibniz, therefore, all mechanical laws might a priori be rephrased in the vocabulary of teleology. This, however, remains without proof. As Daniel Garber remarks, «the idea of parallel explanatory structures everywhere in nature seems a kind of speculative program for a natural philosophy, grounded in a metaphysical vision rather than in detailed argument, empirical or otherwise» (Garber 2009, 259).[26]

Let us now come back to Maupertuis. What does he retain of Leibniz's teleological conception? No doubt that the PLA meets requirement (1). As I shall argue in a further section, Maupertuis considers the PLA the most universal law of nature and, as such, the clearest tangible proof of God's intelligent design. The PLA is the architectonic law *par excellence*, through which the Creator has organised the world, and therefore the cornerstone of the «realm of wisdom». As for requirement (2), things change significantly. Although Maupertuis vindicates the PLA's explanatory power, stating that we should "employ both methods" – «calculate the movements of bodies, but also consult the design of the Intelligence» (Maupertuis 1744, 426) – to come to the right conclusions, a careful analysis of his argument shows, however, that the PLA does not meet requirement (2). The "both-methods" statement remains without consequence, not only in the *Accord*, but in further works too. In the 1744 paper, the PLA is formulated on the basis of the scientific inquiry into the laws of optics (the «realm of power»), but, once formulated, does not seem to have any particular heuristic function. Rather, the PLA is a generalisation, subsuming already-

[25] Quoted in Garber 2009, 258.
[26] For interesting remarks on this point, see Andrault 2016, 107-109.

known laws under a single formulation. All Maupertuis looks for is in fact the agreement between particular mechanical laws and a general metaphysical principle, which is formulated *après coup* in order to make the picture of physical reality more consistent and unitary. Of course, this criticism points to the use Maupertuis *de facto* makes of the PLA: this, one might say, tells us nothing about the *de jure* status of the principle. This I grant. We should, however, note that Maupertuis's ideas on the *de jure* situation, as I showed, are quite confused; the reference to the *de jure* status of the PLA might therefore not be an easy way out of the objection.

In the end, it seems that the main – perhaps the only – application of such a generalisation is the proof of the existence of God Maupertuis formulates by means of the PLA[27].

4. *The* PLA *and the Proof of the Existence of God*

At the outset of the paper *Les lois du mouvement et du repos déduites d'un principe métaphysique* (1746), Maupertuis states that, after discovering the PLA, his present objective is «to draw from this very source some higher and more important truths» (Maupertuis 1746, 267). In section 2 of the *Lois du mouvement*, one finds indeed a detailed presentation of Maupertuis's PLA-based demonstration of the existence of God.

From the very title of the section, one gathers Maupertuis's key assumption: «That the proofs of the existence of God are to be sought in the general laws of nature» (Maupertuis 1746, 277)[28]. This, for a twofold reason: (1) general laws are universal, therefore they do not suffer any exception, and (2) they are simple, therefore unequivocal. The criteria of universality and simplicity, however, should be further specified in order to be actually informative. When can a law be said to be *universal*? It does indeed make a difference if one conceives the method of knowledge to be inductive – in which case, universality is such only *par provision* – or deductive. And then: what are the criteria of *simplicity* for a law? Maupertuis overlooks such definitional questions, and rather focuses on operative ones: *how do we get to* universal and simple laws? He answers by mentioning a discipline that should lead this research, namely mathematics.

[27] I am only referring to Maupertuis's own use of the PLA. Other scientists will develop the mathematical consequences of the PLA and apply it to, e.g., the study of planetary motions (Euler).
[28] An important source for this claim is of course Malebranche.

Why mathematics? Mathematical laws are universal and simple, but also absolutely certain (although not necessary[29]): «Such [mathematical] certainty does not depend on the method the geometers follow, but on the simplicity of the objects they consider» (Maupertuis 1746, 278). While not claiming the irrelevance of method, Maupertuis emphasizes the nature of the object as the decisive criterion to attain certainty – universality, although not mentioned, is closely related to simplicity. The question thus arises of what role the appeal to mathematics plays in this context. There are a few theoretical options on the table. (A) Mathematics might be a mere example from which to draw inspiration: mathematicians produce certain proofs, so should philosophers. (B) The method of mathematics should likewise be used in metaphysics. Although *prima facie* plausible – Christian Wolff, and many eighteenth-century philosophers with him, hold this very position – this option is explicitly discarded by Maupertuis when he says that «[mathematical] certainty does not depend on the method the geometers follow». (C) Mathematical demonstrations and theorems should play some direct role in metaphysics. This is the option Maupertuis endorses. His choice is grounded on a specific epistemological assumption:

> Let us see if we can make a better use of this science [mathematics]. The proofs of the existence of God it will provide should have over all others the advantage of being evident, which is typical of mathematical truths. Those who do not trust metaphysical reasoning will be more at ease with these kind of proofs (Maupertuis 1746, 278).

When Maupertuis states that the existence of God should be proved through mathematics, he seems to presuppose an axiom, one might call the «transitivity of certainty (TOC) axiom». If the demonstration proving that proposition x (mathematical) is true can be used to prove also that proposition y (non-mathematical) is true, then proposition y will hold the same degree of certainty that proposition x does. The axiom works well in all cases where mathematical models are used to demonstrate non-mathematical statements (typically, physical laws). The question Maupertuis leaves unasked, however, is whether the TOC axiom holds even when y is of a wholly different nature than x, e.g., for metaphysical and theological propositions. This appears indeed harder to believe.

[29] Later on in his career, Maupertuis will further reflect on the notions of contingency and necessity: see Charrak 2006.

Leaving aside the problem raised by the TOC axiom, let us come back to Maupertuis's argument, and see how it continues. Maupertuis specifies which mathematical laws attention shall be focused on:

> The Supreme Being is everywhere, but is not everywhere visible to the same extent. We shall better see Him in the simplest objects: let's search for Him in the first laws he imposed on nature […] according to which movement is conserved, distributed, or destroyed (Maupertuis 1746, 279).

This is not surprising, since the PLA is a universal law overruling motion in physical nature. Surprising, however, is what Maupertuis states immediately afterwards, concerning method.

> I could have started from these laws such as they are given by mathematicians, and confirmed by experience, searching therein the marks of God's wisdom and power. However, as those who formulated these laws relied on hypotheses that were not purely geometrical, and therefore their certitude seems not to be established in strict demonstrations, I thought it better and more useful to deduce such laws from the attributes of the almighty and all wise Being. If the laws I find in this way are the same as those we observe in the universe, wouldn't this be the strongest proof that this Being exists, and that He is the author of these laws? (Maupertuis 1746, 279)

The laws of motion, insofar as they are grounded on experience, are not certain in an absolute sense. Mathematics, as all other natural sciences, relies on ancillary hypotheses that are – at least to some extent – arbitrary. Maupertuis insists on this very point in other texts as well, most notably in the *Réflexions philosophiques sur l'origine des langues et la signification des mots* (1740). Here he says: «What we call 'our sciences' depend so closely on the ways in which we first designated perceptions, that I think the questions and propositions would all be different if we had established other expressions out of our first perceptions» (Maupertuis 1768a, 268)[30].

Maupertuis's claim on the uncertainty of the laws of motion, formulated inductively, poses some problems. Mathematical demonstrations would indeed be less strict than purely deductive demonstrations, namely those solely relying on a priori principles. It is therefore hard to see how mathematics could be the norm for the

[30] And also: «I do not think the diversity of their [people of remote countries] philosophy derives from any difference in their first perceptions, but rather from the accustomed language of every country, *from this destination of signs to the different parts of perceptions: destination which is very much arbitrary, and that the first men could have done in so many different ways* […]» (Maupertuis 1768a, 275-276).

strictness of metaphysical demonstrations. (Although indeed Maupertuis discards option (B), options (A) and (C) are still on the table.)

Of particular interest is the method Maupertuis claims he will follow: «I thought it better and more useful to deduce such laws from the attributes of the almighty and all wise Being. If the laws I find in this way are the same as those we observe in the universe […]». Since the inductive formulation of the laws of motion is not absolutely certain, one should rather deduce them top-down, from the divine attributes. If the results of the deduction coincide with those of the induction, then one gets the strongest confirmation of their accuracy. Far from being exhaustive, Maupertuis's explanation creates more confusion than clarity. Indeed, if (bottom-up) induction, in the context of the eighteenth-century Newtonian-inspired empiricist philosophy, is a univocally defined concept, (top-down) deduction is not – this is *a fortiori* the case if deduction is to be made from the attributes of God.

Maupertuis starts with a historical survey of the philosophers' ideas on motion. Some ancient philosophers, confronted with the difficulty of making sense of movement, denied its reality. This is because, Maupertuis says, the only source of knowledge we have about movement is experience, and experience can always be doubtful. Of some propositions, however, we can acquire reliable knowledge by means of reiterated experiences or relevant counterexamples: *e.g.*, motion is not essential to matter, and matter cannot move itself if not impelled by some external cause. The question is not therefore whether matter moves or not, nor whether motion is essential to it, nor whether it can move itself, but what is the external cause causing it to move. Philosophers, Maupertuis says, variously tried to answer the question – appealing to an unmoved mover (Aristotle) or to God (Malebranche) – though unsuccessfully. This because they lacked a fundamental principle, which the laws of motion obey: «It was necessary to know that all laws of motion and rest are grounded on the most suitable principle (*le principe le plus convenable*)[31] to see that they had been established by an all wise and almighty Being» (Maupertuis 1746, 282). In an updated version of the 1746 paper, namely the first part of the *Essai de cosmologie* (1750)[32], Maupertuis further comments: «Therefore, I shall not look for these laws [of motion] in mechanics, but in the wisdom of the Supreme Being» (Maupertuis 1768a,

[31] In the *Essai de Cosmologie* (1750), Maupertuis rather says: «on the principle of the best (*le principe du mieux*)» (Maupertuis 1768a, 35).
[32] Which is otherwise the same text as Maupertuis 1746.

35). Although the only source of knowledge of motion is experience, the principle underlying the very structure of motion is a priori. Maupertuis does not justify this assumption. To do so, the only argument he could adduce is a circular one: laws of motion are grounded on the principle of the best because God is their author, the strongest proof that God is their author being that they are grounded on the principle of the best.

Pursuing his discussion, Maupertuis notes that modern authors – Descartes, Huygens, and Leibniz – were not much bothered by discussions on the nature of motion, but rather focused on the laws according to which «it is distributed, conserved, and destroyed» (Maupertuis 1746, 283). After mentioning the debate over the existence of hard bodies[33], Maupertuis discusses Descartes's and Leibniz's opposing views on the conservation of movement. Descartes argued that the quantity of movement is constant in nature, whereas Leibniz held that living force (measured by the product of the mass and the square of velocity) is a constant. Yet, Maupertuis contends, «conservation of movement is true only in some cases, [and] conservation of force takes place only for certain bodies. Neither of them can be considered a universal principle, nor a general result (*un résultat général*) of the laws of motion» (Maupertuis 1746, 285). Maupertuis himself is the first who has in fact discovered the truly universal principle of motion, namely «the *principle of least action*: so wise a principle, and so worthy of the Supreme Being» (Maupertuis 1746, 286). From here, Maupertuis concludes: «The laws of movement and rest deduced from this principle are precisely the same as those we observe in nature» (Maupertuis 1746, 286).

The conclusion of the argument is therefore that the laws of movement one finds through empirical investigation are the same as those one can deduce from the PLA. The PLA thus plays the role of the *explanans*, rather than of the *explanandum*. In other words, the PLA is not the law that is to be deduced, but a most general principle from which to deduce particular laws. But how is this principle found? This Maupertuis leaves unsettled. To begin with, no deduction from the attributes of God – whatever this might mean – takes place. Two alternatives then remain: if the PLA is found intuitively, it might count as a first-type principle, and be the cornerstone of some «superior science»; if on the contrary it is found inductively, it would be a second-type principle, more general indeed than all other second-type principles – not

[33] On this debate, see Shea 1988.

absolutely general and universal, though. Maupertuis hesitates over these two alternatives, none of which is fully and satisfactorily argued.

Conclusion

In this paper, I have tried to highlight some of the difficulties to be found in Maupertuis's writings (1740-1746) on PLA. It emerges from my analysis that the epistemological status of PLA is problematic insofar as Maupertuis tends to present it as a universal principle of nature even though this claim is incompatible with the empiricist inspiration of his philosophy. While Maupertuis's idea of scientific method is mostly borrowed from Newton, the influence of the Leibnizian tradition pushes him toward architectonic reasoning: this creates a tension in his argument that cannot be easily resolved.

REFERENCES

PRIMARY SOURCES

d'Alembert, Jean Le Rond. 1751. «Action». In *Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers,* Tome 1, 119-120. Paris: Briasson *et al*.

Leibniz, Gottfried W. 1989. *Philosophical Papers and Letters*. Ed. by Leroy E. Loemker. Dordrecht: Kluwer.

Maupertuis, Pierre-Louis Moreau de. 1740. «Lois du repos des corps». *Mémoires de l'Académie Royale des Sciences*: 170-176.

–––. 1744. «Accord de différentes loix de la nature qui avaient jusqu'ici paru incompatibles». *Mémoires de l'Académie Royale des Sciences*: 417-426.

–––. 1746. «Les loix du mouvement et du repos, déduites d'un principe de métaphysique». *Mémoires de l'Académie Royale des Sciences et Belles-Lettres de Berlin*: 267-294.

–––. 1756. «Examen philosophique de la preuve de l'existence de Dieu employée dans l'*Essai de cosmologie*». *Mémoires de l'Académie Royale des Sciences et Belles-Lettres de Berlin*: 389-424.

–––. 1768a. *Œuvres*. Tome 1. Lyon: Bruyset

——. 1768b. *Œuvres*. Tome 2. Lyon: Bruyset.

——. 1768c. *Œuvres*. Tome 4. Lyon: Bruyset.

Newton, Isaac. 1999. *The* Principia. *Mathematical Principles of Natural Philosophy*. Ed. by I. Bernard Cohen and Anne Whitman. Berkeley: University of California Press.

SECONDARY LITERATURE

Andrault, Raphaële. 2016. *La raison des corps. Mécanisme et sciences médicales*. Paris: Vrin.

Bachelard, Suzanne. 1961. *Les polémiques concernant le principe de la moindre action au XVIII^e^ siècle*. Paris: Palais de la découverte.

Beeson, David. 1992. *Maupertuis: An Intellectual Biography*. Oxford: The Voltaire Foundation.

Belkind, Ori. 2017. «On Newtonian Induction». *Philosophy of Science* 84 (4): 677-697.

Brown, Harcourt. 1963. «Maupertuis *philosophe*: Enlightenment and the Berlin Academy». In *Transactions of the First International Congress on the Enlightenment*, ed. by Theodore Besterman, 255-269. Geneva: Institut et Musée Voltaire.

Brunet, Pierre. 1929. *Maupertuis. L'œuvre et sa place dans la pensée scientifique et philosophique du XVIII^e^ siècle*. Paris: Blanchard.

——. 1938. *Étude historique sur le principe de la moindre action*. Paris: Hermann.

Casini, Paolo. 1983. *Newton e la coscienza europea*. Bologna: Il Mulino.

Charrak, André. 2006. *Contingence et nécessité des lois de la nature au XVIII^e^ siècle. La philosophie seconde des Lumières*. Paris: Vrin.

Dolghi, Vlad. 2017. «The Role of Mathematics in Maupertuis's Epistemology and Natural Philosophy». *Society and Politics* 11 (1): 26-45.

Duchesneau, François. 1993. *Leibniz et la méthode de la science*. Paris: Presses universitaires de France.

Ducheyne, Steffen. 2012. *«The Main Business of Natural Philosophy». Isaac Newton's Natural-Philosophical Methodology*. Dordrecht: Springer.

Fee, Jerome. 1941. «Maupertuis, and the Principle of Least Action». *The Scientific Monthly* 52 (6): 496-503.

Feher, Maria. 1988. «The role of metaphor and analogy in the birth of the principle of least action of Maupertuis (1698–1759)». *International Studies in the Philosophy of Science* 2 (2): 175-188.

Garber, Daniel. 2009. *Leibniz: Body, Substance, Monad*. Oxford: Oxford University Press.

Goldenbaum, Ursula. 2016. *Ein gefälschter Leibnizbrief? Plädoyer für seine Authentizität*. Hannover: Wehrhahn Verlag.

Kneser, Adolf. 1929. *Das prinzip der kleinsten Wirkung von Leibniz bis zur Gegenwart*. Leipzig and Berlin: Teubner.

Leduc, Christian. 2015. «La métaphysique de la nature à l'Académie de Berlin». *Philosophiques* 42 (1): 11-30.

Lyssy, Ansgar. 2015. «L'économie de la nature – Maupertuis et Euler sur le principe de moindre action». *Philosophiques* 42 (1): 31-50.

McDonough, Jeffrey K. 2009. «Leibniz on Natural Teleology and the Laws of Optics». *Philosophy and Phenomenological Research* 78 (3): 505-544.

Panza, Marco. 1995. «De la nature épargnante aux forces généreuses: le principe de moindre action entre mathématiques et métaphysique. Maupertuis et Euler, 1740-1751». *Revue d'histoire des sciences* 48 (4): 435-520.

Pulte, Helmut. 1989. *Das Prinzip der kleinsten Wirkung und die Kraftkonzeptionen der rationalen Mechanik: Eine Untersuchung zur Grundlegungsproblematik bei Leonhard Euler, Pierre Louis Moreau de Maupertuis und Joseph Louis Lagrange*. Stuttgart: Franz Steiner Verlag.

Radelet de Grave, Patricia. 1998. «La moindre action comme lien entre la philosophie naturelle et la mécanique analytique». *Revista de la Sociedad Española de Historia de las Ciencias y de las Técnicas* 21: 439-484.

Sabra, Abdelhamid I. 1981. *Theories of Light From Descartes to Newton*. Cambridge: Cambridge University Press.

Shea, William R. 1988. «The Unfinished Revolution: Johann Bernoulli (1667-1748) and the Debate Between the Cartesians and the Newtonians». In *Revolutions in Science*, ed. by William R. Shea, 70-92. Canton, MA: Science History Publications.

Terrall, Mary. 2002. *The Man Who Flattened the Earth. Maupertuis and the Sciences in the Enlightenment*. Chicago: The University of Chicago Press.

Tonelli, Giorgio. 1987. *La pensée philosophique de Maupertuis: son milieu et ses sources*. Hildesheim: Georg Olms.

Van den Abbeel, Yannick. 2017. «The Tension between the Mathematical and Metaphysical Strands of Maupertuis' Principle of Least Action». *Noctua* 4 (1/2): 56-90.